

Study on Dynamic of Giant Magnetostrictive Material Transducer with Spring of Nonlinear Stiffness

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Abstract

To study the dynamics of giant magnetostrictive material (GMM) transducer, its model was developed, according to the dynamic experiment results of GMM and the influence of unsymmetrical piece-wise linear stiffness of the pre-pressing spring. Based on the one degree of freedom its vibration model of a GMM transducer, unsymmetrical piece-wise linear nonlinear characteristic of pre-pressing spring was investigated. The first order harmonic motion component of the GMM transducer was obtained by the analysis of KBM method. By the numerical simulation the complicated bifurcation and chaos behavior of the nonlinear vibration system were founded, which should to be taken account of in the design of GMM transducer.

Keywords: Giant magnetostrictive material transducer; Nonlinear stiffness

1. Introduction

It is well known that the length of some ferrous materials will change under the effect of magnetic fields, which is called magnetostriction. At 1970's, GMM was discovered whose magnetostriction coefficient is hundred times more than that of non rare earth ferroalloys at room temperature. Among the superior properties of GMM are large magnetostriction coefficient, strong magnetomechanical coupling, huge output mechanical stress and energy density, as well as immediate mechanical response to excitement (Zhu et al., 2002). Therefore it is well suitable for the devices requiring high power, small structure size and mass, high accuracy but micro displacement. And it is very useful in manufacture of micro actuators, controllers, supersonic energy exchanger and mechanical processor, geological pros-

pecting, as well as nondestructive inspecting.

The need for transducers is increasing in defense as well as civilian applications. It is known that GMM using rare earth and iron have properties well suitable for them to be employed as active elements in very low frequency, high power transducers with a considerable reduction in size and shape compared to the piezoceramic transducers (Kyarnsjo et al., 2000; Li and Yuan, 2003).

In this paper, according as experiment the equivalent mechanics model of a GMM transducer was established. By the analysis of KBM method, the first order harmonic motion component of the GMM transducer was obtained.

2. Model of the GMM transducer

The schematic diagram of the GMM transducer is illustrated in Fig. 1. The GMM rod is surrounded by a solenoid-coil through which an alternating current passed. The GMM rod can be elongated or shortened as the intensity of magnetic fields produced by the

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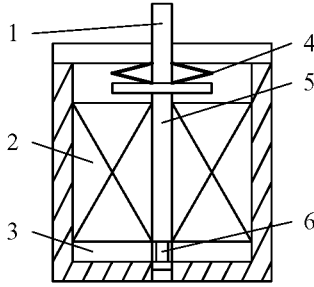


Fig. 1. Sketch of GMM transducer.

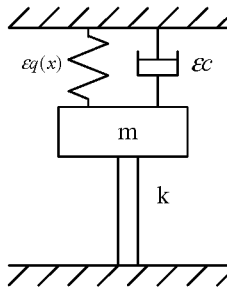


Fig. 2. The dynamic model of GMM transducer.

solenoid-coil becomes strong or weak. The GMM rod is brittle in nature and can fracture under high dynamic operation. To prevent this, the GMM rod is repressed with pre-pressing spring. In addition, the performance of GMM improves under the application of a compressive mechanical bias (Anjanappa, 1994; Nakamura, et al., 2001).

The dynamic model of GMM transducer is shown in Fig. 2. Suppose that m is mass, εc is damping factor, $\varepsilon q(x)$ is nonlinear stiffness of pre-pressing spring, k is stiffness of GMM rod, $\varepsilon F \sin(\omega t)$ is force produced by magnetic fields that acted on GMM rod, and ignore the effect of other factors, so that the motion differential equation of GMM transducer are given by

$$m\ddot{x} + \varepsilon c\dot{x} + kx + \varepsilon q(x) = \varepsilon F \sin(\omega t) \quad (1)$$

Where

$$\varepsilon q(x) = \begin{cases} k_1 x + \alpha_1 & x < \xi \\ k_2 x + \alpha_2 & x \geq \xi \end{cases} \quad (2)$$

3. Analysis for vibration system with piece-wise linearity

The motion differential equation of GMM transducer with normal form is as follows.

$$\begin{aligned} m\ddot{x} + kx &= \varepsilon f(x, \dot{x}) + \varepsilon F \sin(\omega t) \\ \varepsilon f(x, \dot{x}) &= -\varepsilon cx - \varepsilon q(x) \end{aligned} \quad (3)$$

Suppose that the first order harmonic motion component is as follows.

$$\begin{aligned} x &= a \cos \varphi \\ \varphi &= \omega t + \phi \end{aligned} \quad (4)$$

For ξ , corresponding phasic angle $\varphi_0 = \arccos(\xi/a)$ and ϕ meet

$$\begin{cases} \dot{a} = -\delta_e(a)a - \frac{\varepsilon F}{m(p_0 + \omega)} \cos \phi \\ \dot{\phi} = p_e(a) - \omega - \frac{\varepsilon F}{m(p_0 + \omega)} \sin \phi \\ k_e(a) = k + \frac{\varepsilon}{\pi a} [k_2 a \pi + a \varphi_0 (k_1 - k_2) + \frac{1}{2}(k_1 - k_2)a \sin 2\varphi_0 + 2(a_1 - a_2) \sin \varphi_0] \\ \delta_e(a) = \varepsilon c / (2m) \\ p_e(a) = \sqrt{k_e(a) / m} \\ p_0 = \sqrt{m/k} \end{cases} \quad (5)$$

Where p_0 is natural frequency, $p_e(a)$ is equivalent natural frequency, $\delta_e(a)$ is equivalent damping ratio and $ke(a)$ is equivalent stiffness.

Letting \dot{a} and $\dot{\phi}$ in Eq. (5) are zero, equation of stationary motion is derived as follows.

$$\begin{cases} 2m\omega a \delta_e(a) = -\varepsilon F \cos \phi \\ ma[p_e^2(a) - \omega^2] = -\varepsilon F \sin \phi \end{cases} \quad (6)$$

And from Eq. (6), following frequency-response equation is expressed by

$$m^2 a^2 \{ [p_e^2(a) - \omega^2]^2 + 4\omega^2 \delta_e^2(a) \} = \varepsilon^2 F^2 \quad (7)$$

4. Bifurcation and chaos analysis for vibration system with piece-wise linearity

For Eq. (1), inducing transformation non-dimension

$$\begin{aligned} \frac{k+k_1}{m} &= \omega_1^2, \frac{k+k_2}{m} = \omega_2^2, \frac{\omega_1}{\omega_2} = l, y = \frac{x}{\xi}, \omega_1 t = \tau, \frac{\omega}{\omega_1} = \varpi, \quad (8) \\ 2\zeta &= \frac{\varepsilon c}{m\omega_1}, p = \frac{\varepsilon F}{m\xi\omega_1^2}, \beta_1 = \frac{\alpha_1}{m\xi\omega_1^2}, \beta_2 = \frac{\alpha_2}{m\xi\omega_1^2} \end{aligned}$$

the motion differential equation of transducer with non-dimension is as follows.

$$\begin{aligned} \ddot{y} + 2\zeta\dot{y} + f(y) &= p \sin \varpi \tau \\ f(y) &= \begin{cases} y + \beta_1 & y \leq 1 \\ l^2 y + \beta_2 & y > 1 \end{cases} \end{aligned} \quad (9)$$

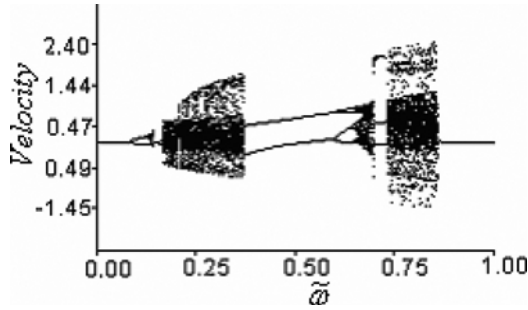


Fig. 3. Bifurcation diagram of the piecewise-linear system.

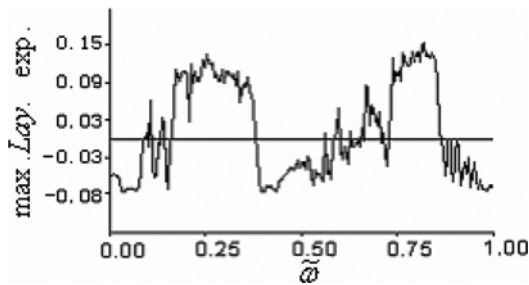


Fig. 4. Max. Lyapunov exponent diagram of the piecewise-linear system.

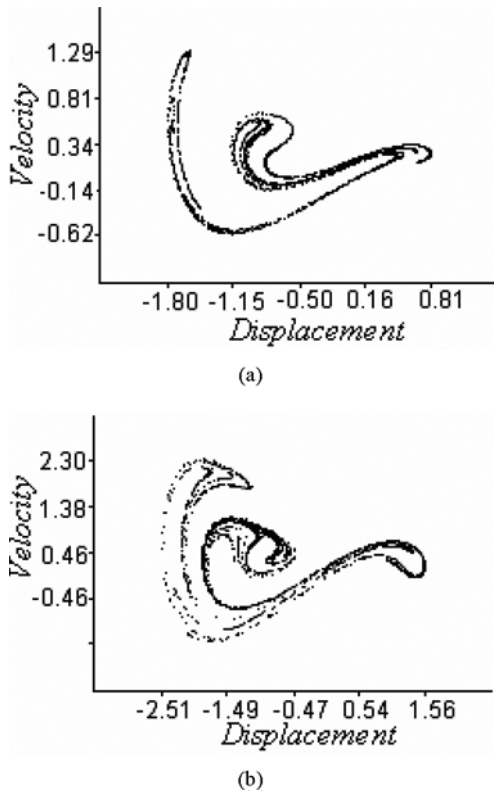


Fig. 5. Dynamic behavior of system at $\omega = 0.25$ and $\omega = 0.78$.

Letting $p=1.5$, $\zeta=0.02$, $l=2.0$ and $\beta_1=0$, therefore pre-pressing spring is hardening spring. The results of numerical simulation are illustrated from Figs. 3, 4 and 5(a), (b). Figures 3 and 4 are bifurcation diagram and max. laypunov exponent diagram of the piecewise linear system with frequency ratio $\omega = 0 \sim 1.0$, respectively. It is known from Figs. 3 and 4 that at frequency ratio of $\omega = 0.173$ the orbit of the piecewise-linear system enters chaos by explosive bifurcation, at $\omega = 0.357$ the orbit of the piecewise-linear system is off from chaos by contradictorily bifurcation, at $\omega = 0.59$ the orbit of the piecewise-linear system enters chaos by pitchfork bifurcation, at $\omega = 0.871$ the orbit of the piecewise-linear system is off from chaos.

Figures 5(a) and (b) Poincare map and phase trajectory diagram at $\omega = 0.25$ and $\omega = 0.78$, respectively. It is known from these figures that the orbit of the piecewise linear system is in chaos.

5. Conclusions

In this paper, taking into account the influence of unsymmetrical piece-wise linear stiffness of the pre-pressing spring, the equivalent mechanics model of a GMM transducer was established. Base on the one-freedom-degree vibration model of a GMM transducer, unsymmetrical piece-wise linear nonlinear characteristic from pre-pressing spring was investigated. By the numerical simulation, the complicated bifurcation and chaos behavior of the vibration system with the unsymmetrical piece-wise linear were founded. Sometimes, it is useful in the design of GMM transducer, for instance, GMM transducer purposely designed in chaos state can yields wide frequency band response that is necessary to the mineral exploration.

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References

Zhu houqing, liu Jianguo, Wang Xiurong, Xing Yanhong and Zhang HongPing, 2002, "Applicatins of Terfenol-D in China," *Journal of Alloys and Compounds*, Vol. 68, No. 6, pp.49~55

Kvarnsjo and Engdahl G, 2000, "A Set-Up for Dynamic Measurements of Magnetic and Mechanical Behavior of Magnetostrictive Materials," *IEEE Transactions on Magnetics*, Vol. 25, No. 5, pp. 4195~4203

Li He and Yuan Huiqun, 2003, "Analysis on Nonlinear Dynamic of Magnetostrictive Actuator." *Journal of Nonlinear Dynamics in Science and Technology*, Vol. 8, No. 2, pp.153~159

Anjanappa, M. and Bij, 1994, "A Theoretical and Experimental Study of Magnetostrictive Mini-actuator," *Smart Material Structure*, Vol. 23, No. 3, pp.83~95

Nakamura, T., Nakano, I., Kawamoria et al., 2001, "Static and Dynamic Characteristics of Giant Magnetostrictive Materials Under High Pre-stress," *Journal of Applied Physics*, Vol. 40, No. 5, pp.3658~3673